EAST ALPINE MEETING ON DIFFERENTIAL EQUATIONS AND DYNAMICAL SYSTEMS

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ABSTRACTS

OPEN SETS OF EXPONENTIALLY MIXING ANOSOV FLOWS

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If a flow is sufficiently close to a volume-preserving Anosov flow and $\dim E_s = 1$, $\dim E_u \geq 2$ then the flow mixes exponentially whenever the stable and unstable bundles are not jointly integrable (similarly if the requirements on stable and unstable bundle are reversed). This implies the existence of non-empty open sets of exponentially mixing Anosov flows. (Joint work with Khadim War).

GLOBAL ATTRACTORS AND THEIR STRUCTURE FOR SCALAR DELAY DIFFERENCE EQUATIONS

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We examine the existence and structure of the global attractor of the following class of difference equations

$$x_{k+1} = f(x_k, x_{k-d}).$$

Under appropriate assumptions on f, we give a so-called Morse decomposition of the global attractor. This decomposition is based on an integer valued Lyapunov function introduced by J. Mallet-Paret and G. Sell [2] as a discrete time counterpart of their celebrated discrete Lyapunov function for delay differential equations [1]. Our results apply e.g. to several discrete-time models arising from life-sciences, such as Ricker's equation, May's genotype model, the Wazewska–Lasota equation or the discrete Mackey–Glass equation.

This is a joint work with Christian Pötzsche (Alpen-Adria Universität Klagenfurt).

References

[1] J. Mallet-Paret and G. R. Sell, Systems of differential delay equations: Floquet multipliers and discrete Lyapunov functions, *J. Differential Equations* **125** (1996), no. 2, 385–440.
[2] J. Mallet-Paret and G. R. Sell, Differential systems with feedback: time discretizations and Lyapunov functions, *J. Dynam. Differential Equations* **15** (2003), no. 2–3, 659–698.

CONTROLLABILITY AND PUGH'S CLOSING LEMMA FOR SOME FLOWS WITH INFINITE INVARIANT MEASURES

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In a recent paper by D. Burago, S. Ivanov and A. Novikov, [1], it has been shown that a fish with limited velocity capabilities can reach any point in the (possibly unbounded) ocean provided that the fluid velocity field is incompressible, bounded and has vanishing mean drift. This brilliant result substantially extends the celebrated point-to-point controllability theorems though being in a sense non constructive. We will give a fish the constructive recipe of how to survive in a turbulent ocean, and show how this is related to closing lemmas for dynamical systems, in particular to C. Pugh's closing lemma, proving an extension of the latter to incompressible vector fields over a possibly unbounded domain.

This is a joint work with Eugene Stepanov (Saint-Petersburg Department of Steklov Mathematical Institute, Saint-Petersburg, Russia).

Reference

[1] D. Burago, S. Ivanov and A. Novikov, A survival guide for feeble fish, Algebra i Analiz, **29** (2017) 49–59.

PHYSICAL MEASURES FOR DYNAMICAL SYSTEMS

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We present the Palis and Viana conjectures on the existence of physical measures, review some of the literature, and describe some recent joint work with Climenhaga and Pesin on the existence of Sinai-Ruelle-Bowen physical measures for nonuniformly hyperbolic surface diffeomorphisms.

ON NONAUTONOMOUS BIFURCATIONS

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We discuss several approaches to develop a bifurcation theory for nonautonomous differential or difference equations. This includes attractor and solution bifurcations, as well as the related spectral theory.

Reference

[1] Bifurcations in nonautonomous dynamical systems: Results and tools in discrete time, in Proceedings of the international workshop "Future Directions in Difference Equations", June 13-17 2011, Vigo, Spain, 163-212.
`http://www.dma.uvigo.es/~eliz/pdf/Potzsche.pdf`

INVARIANT SURFACES AND FIRST INTEGRALS OF THE MAY-LEONARD ASYMMETRIC SYSTEM

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We investigate existence of first integrals in the three dimensional May-Leonard asymmetric system. Using the computational algebra systems Mathematica and Singular we first look for subfamilies of the May-Leonard asymmetric system admitting invariant surfaces of degree one and two. Then using these invariant surfaces we identify subfamilies of the system admitting analytic first integral.

References

[1] V. Antonov, D. Dolicanin, V. G. Romanovski, J. Toth, MATCH Commun. Math. Comput. Chem. **76** (2016) 455–474.
[2] V. Antonov, W. Fernandes, V. G. Romanovski, N. L. Shcheglova, First integrals of the May-Leonard asymmetric system, 2016, submitted.

ε-NEIGHBORHOODS OF ORBITS OF DYNAMICAL SYSTEMS

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In this talk I give the results concerning analysis of ε-neighborhoods of orbits of dynamical systems, obtained by scientific group at University of Zagreb and the collaborators. The idea for that analysis comes from the fractal geometry, while the motivation comes from the 16th Hilbert problem. The problem asks for an upper bound of the number of limit cycles of polynomial vector fields in the plane, as a function of the degree of the vector field. It is of interest to determine how many limit cycles can bifurcate from a given limit periodic set in a generic unfolding. This number is called the cyclicity of the limit periodic set. The cyclicity is classically obtained by studying the multiplicity of fixed points of the Poincaré map. We establish a relation between the cyclicity of a limit periodic set of a planar system and the leading term of the asymptotic expansion of area of ε-neighborhoods of the Poincaré map of an orbit. A natural idea is that higher density of orbits reveals higher cyclicity. The box dimension, could be read from the leading term of the asymptotic expansion of area of ε-neighborhood.

It was discovered that the box dimension of an orbit signals the bifurcation. The generic bifurcations of 1-dimensional discrete systems are characterized by the box dimension of orbits. Hopf and Hopf-Takens bifurcations could be studied using bifurcation of discrete system defined by the Poincaré map. The Poincaré map near weak focus and limit cycle is differentiable, which is crucial for relating the box dimension and the cyclicity of a limit periodic set.

The problem in hyperbolic polycycle case is that the Poincaré map, usually called Dulac map, is not differentiable. We introduce the appropriate generalizations of box dimension, depending on a particular scale for a given problem. The cyclicity was concluded in the case of saddle loops. If we study more terms in the asymptotic expansion of area of ε-neighborhood, we can obtain more information about the Dulac map, including formal normal forms and formal classification.