


The 6th Symposium on Hypercompositional Algebra new Developments and Applications

Book of Abstracts

May 2026

UNIVERZA V NOVI GORICI
SCIENTIA 1995 VINCES

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A Functorial Radical Approximation for Krasner Hypermodules and Its Topological Correspondence

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Abstract

This study develops a functorial, radical-based framework in the category of Krasner hypermodules over a Krasner hyperring. We introduce idempotent subfunctors of the identity, regarded as radicals, and investigate their hyperstructural properties, including heredity, stability under morphisms, and exactness. We further establish a Gabriel-type correspondence for Krasner hyperrings, thereby linking these functorial structures with an appropriate notion of topology. A range of illustrative examples, including constructions over hyperdomains and -adic sets, is presented to demonstrate the breadth and applicability of the context. These results provide a coherent categorical and homological perspective on Krasner hypermodules and open new avenues for further research in hypermodule theory.

HSP Theorem for Morgado Hyperlattices

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Abstract

This work investigates the categorical and algebraic properties of Morgado hyperlattices, a class of hyperstructures introduced by J. Morgado. Here we have expanded the analysis started by Marcelo E. Coniglio, Ana Claudia Golzio, and Kaique Matias de Andrade Roberto, by providing alternative descriptions of these structures as hyperalgebras and establish a systematic study of the category MHL (Morgado Hyperlattices) and its variants. A central part of our analysis involves the introduction of a homotopy (similarity) relation between morphisms, leading to the quotient category hMHL. We demonstrate that the category of homotopic Morgado hyperlattices with bottom elements, $\mathsf{hMHL}_{\perp}$, is both complete and cocomplete, providing explicit constructions for products, pullbacks, equalizers, and their duals. Furthermore, we develop non-deterministic versions of categorical constructions, such as kernels and coequalizers, to account for the multi-valued nature of the operations. The main result of this research is the adaptation of classical Birkhoff's HSP Theorem to the context of Morgado hyperlattices. We characterize varieties as classes of hyperlattices closed under homomorphic images (H), sub-hyperlattices (S), and direct products (P), offering specific variations for categories defined by full and pre-order reflecting morphisms. These results contribute to the broader program of non-deterministic algebraization of logics and the study of generalized algebraic categories.

On algebraic closure of superfields

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Abstract

The concept of hyperrings was introduced by Marc Krasner in the 1957. The concept of polynomials (and its roots) over hyperrings are recently. In the present talk, we expand the basic theory of algebraic extensions to the realm of superfields (a field with multivalued operations of sum and product), showing that every superfield has a (unique up to isomorphism) full algebraic extension to a superfield that is algebraically closed. Moreover, we show that every infinite algebraically closed superfield admits a quantifier elimination procedure that, in some sense corresponds to a model-theoretic statement which cannot be the traditional quantifier elimination.

Commutativity Measures and Graph Structures Associated with Finite Groups

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Abstract

The interaction between finite group theory and graph theory continues to generate powerful methods for interpreting algebraic phenomena through combinatorial structures. In particular, graph models derived from group properties offer intuitive and computationally tractable frameworks for examining internal symmetries and element relationships. This talk centers on quantitative measures of commutativity in finite groups and their influence on the formation of associated graph structures. Various formulations of commutativity degree are examined, with emphasis on how these probabilistic measures capture essential features of a group's algebraic behavior. Guided by these measures, we develop graph constructions in which vertices correspond to group elements and subgroups, while adjacency is defined through commutation conditions and related interaction criteria. The resulting graphs are investigated from both structural and spectral perspectives, and representative families of finite groups are presented as illustrative case studies. Overall, this work highlights the role of commutativity measures not only as numerical invariants but also as generative tools for meaningful graph models, further reinforcing the connection between probabilistic group theory and combinatorial graph analysis.

The Category of Krasner Hypermodules and Exact Short Sequences

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Abstract

Let R be a Krasner hyperring. By $\mathsf{HypMod}(R)$ we denote the category of Krasner R-hypermodules. It is known that it is an exact category, which is not additive. In this talk, we study on some homological objects of $\mathsf{HypMod}(R)$ via short exact sequences.

Order-Generated Hyperrings and Associated Multilattices

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Abstract

We present a class of finite order-generated hyperrings where hyperaddition and hypermultiplication generate set-valued join and meet operations compatible with multilattices. We provide formal definitions, show that these operations satisfy multilattice axioms in the sense of Benado, and illustrate the theory with several concrete examples, including tables and Hasse diagrams. We also discuss the notions of ideals and filters in these structures, and show that different hyperrings can induce the same multilattice.

One Axiom Fewer: A Reduction of the Axiomatic Basis for Hypergroups, Hyperfields, Hypermodules, and Related Structures

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Abstract

The independence of axioms is a foundational concern in mathematics, with profound implications for the logical architecture of entire theories. This paper examines the axiomatic basis of several core structures in Hypercompositional Algebra and demonstrates that a number of axioms traditionally included in their definitions are, in fact, derivable from the remaining ones and therefore need not be postulated independently. For hypergroups, it is shown that the requirement that the hypercomposition of any two elements be non-empty follows directly from associativity and reproductivity, rendering it redundant as an axiom. This result extends by corollary to all structures built upon hypergroups, including hyperrings, hyperfields, hypermodules, and vector hyperspaces. The same redundancy is established for left and right almost-hypergroups, HV-groups, and multiplicative hyperrings. In the case of M-polysymmetrical hypergroups, it is proved that the axiom of reversibility is a consequence of the remaining defining axioms, thereby reducing the number of independent postulates by one. An analogous result is established for hyperfields and unitary hyperrings: reversibility follows necessarily from distributivity, via the identity $-(x+y) = -x-y$, and may therefore be omitted from their definitions. This simplification carries over to hypermodules and vector hyperspaces, where the requirement that the additive structure be a canonical hypergroup is shown to be derivable rather than primitive. It is worth noting, however, that this reduction does not extend to non-unitary hyperrings: the standard Dorroh extension fails to embed a hyperring into a hyperring with identity, since the resulting multiplication is not associative and the structure does not even qualify as a superring. The proposed axiomatic reductions yield strictly more minimal definitions in which every retained axiom is genuinely independent. Beyond their theoretical significance, these refinements have direct computational implications: minimizing the axiom set reduces the number of conditions that must be verified algorithmically, enabling more efficient construction and classification of finite hypercompositional structures. This methodology was instrumental in the recent complete enumeration of all hyperfields of order seven.

Hyperalgebraic semantics based on Hyper Boolean Algebras: applications to LFIs

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Abstract

We recently introduced the notion of hyper swap structures, a novel class of hyperalgebras that naturally generalize swap structure semantics. In this talk we introduce the concept of hyper Boolean algebras based on Morgado hyperlattices, showing some basic properties. After this, we show that several paraconsistent logics in the hierarchy of Logics of Formal Inconsistency (LFIs) can be naturally characterized in terms of hyper swap structures semantics generated by hyper Boolean algebras. This is a joint work with K. Roberto and A.C. Golzio.

Strongest Strong Paths and Hypercompositional Structures in Fuzzy Directed Graphs

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Abstract

This talk presents a path-based algebraic framework for fuzzy directed graphs built on the notion of strongest strong paths. Two fuzzy hyperoperations are introduced, one on vertices and one on edges, capturing graded connectivity through structurally optimal routes. The presentation highlights the associativity of the vertex hyperoperation, the induced fuzzy hypersemigroup structure, the role of full-strength connectivity classes, the construction of quotient fuzzy graphs, and the associated fuzzy distance function. Applications to trust-aware systems, communication networks, and biological signaling models illustrate how the framework supports structural abstraction and uncertainty-aware network analysis.

Multivalued Algebraic Structures and Some Genetic Inheritance Models

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(Joint work with K.G. Ilori and T.G. Jaiyeola)

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Abstract

This research establishes a rigorous algebraic framework for modeling genetic inheritance using multivalued algebraic structures. We present distinct multivalued structures that model genetic inheritance across various scenarios: autosomal linkage, epistasis, sex linkage in plants (dioecious) and animals (XY system), and humans (X-linked recessive/dominant). Each multivalued Cayley table defines a hyperoperation $x * y =$ set of possible genotypes of the offspring. We determine whether each satisfies the hyperquasigroup axioms or forms other multivalued algebraic structures (e.g., a commutative hypergroup, a polygroup, or a hypergroupoid). Algebraic properties including flexibility ($x * (y * x) = (x * y) * x$), power-associativity, left alternative ($x * (x * y) = (x * x) * y$), and right alternative ($(y * x) * x = y * (x * x)$) are evaluated.

Analysis of Weak Associativity Properties of Hypercompositional Algebraic Structures in ABO Blood Groups

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Abstract

This paper expounds on algebraic analysis of weak associative hypercompositional structures to incorporate biological inheritance, including the ABO blood group system coupled with the Rh factor. This study aims to use nuclearity, flexibility, and alternativity, and quantify new weak associative properties termed Moufang I and Moufang II identities to evaluate (using computational means) the hypercompositional structures on biological patterns of inheritance. The blood groups A and B of the ABO blood group met the requirements of Moufang I and II identities with a probability of 1.00, which ensures that the inheritance is symmetric, whereas type AB does not. The ABO blood group inheritance was found to possess the property of flexibility owing to the value of probability of 1.00; thus, this blood type is a stable and reversible inheritance. Other blood types, A^+, A^-, B^+, and B^-, exhibited the probability value of 1.000 for the Moufang I and II identities. These findings emphasize the weak associative nature of algebraic hypercompositional structures to model biological inheritance and demonstrate how genetic symmetry and stability can be reflected by algebraic properties. Similarly, the ABO system and Rh factor inheritance fulfilled the flexibility property because of the probability value of 1.00. The study found out that the blood types that are the least prevalent (based on probability) are AB, AB^+, and AB^-, which confirms the already existing results that AB, AB^+, and AB^- are the least common blood types. The distribution of measure of weak associativity (MWA)-probabilities for ABO blood group inheritance, exhibited the chain $0.5156 < 0.5469 < 0.5625 < 0.6563 < 1$, while the distribution of MWA-probabilities for dihybrid cross of ABO system and Rh factor inheritance exhibited the chain $0.500 < 0.625 < 0.6406 < 1.000$. The results are indicative of the wider applicability of the theory of algebraic hypercompositional structure in genetics and indicate the future research directions in interdisciplinary research.

On the Algebraic Structure of the Lattice of Convex Subhyperlattices

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Abstract

Nayak et al. (2025) introduced the concept of convex subhyperlattices of a strong join hyperlattice $\mathfrak{L}$ and demonstrated that the collection of all convex subhyperlattices, $CS(\mathfrak{L})$, together with empty set, forms a lattice. In particular, for P-hyperlattices and Nakano hyperlattices, they obtained a unique representation of convex subhyperlattices in terms of hyperideals and hyperfilters. In this work, we investigate the structure of convex subhyperlattices in join hyperlattices. It is shown that the collection $CS(\mathfrak{L}) \cup \{\emptyset\}$, under set inclusion, forms an algebraic lattice, extending earlier results on its lattice structure. We introduce two new order relations on $CS(\mathfrak{L})$ and study their fundamental properties. In particular, we establish conditions under which these orders are equivalent. Furthermore, we provide a characterization of convex subhyperlattices in P-hyperlattices in terms of initial and final segments. Several examples are included to illustrate the results and to highlight differences that arise in the general setting.

An Overview of Cyclic Codes Through the Lens of Hypercompositional Algebra

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Abstract

The interplay between classical algebraic structures and coding theory has led to significant advance- ments in data transmission. This paper explores the transition from traditional cyclic codes, defined over polynomial rings, to the emerging framework of hypercompositional algebra. We review the fundamental properties of cyclic codes over finite fields and discuss the formal replacement of the standard addition with a multi-valued hyperoperation. By examining the structure of polynomial hyperrings, we provide a conceptual bridge between the two fields, highlighting how hyper-ideals can generalize the properties of generator polynomials. This approach aims to provide a unified perspective on algebraic coding, offering a versatile foundation for studying codes with complex structural constraints without losing the computational advantages of cyclic shifts.

(m,n)-Hyperideals and (m,n)-Quasi-Hyperideals in Ordered Semihypergroups

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Abstract

This study focuses on (m,n)-hyperideals and (m,n)-quasi-hyperideals in ordered semihypergroups. It introduces generalized hyperideal structures, examines their properties, and characterizes regularity conditions. Further, it explores minimal quasi-hyperideals, associated relations, and quasi-hypersimple structures. The results provide a unified framework for analyzing generalized ideal structures in hyperstructure theory.

A parallel between power rings and HX-rings

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Abstract

The group structure has been upgraded from the underlying set to its power set, defining the so called HX-group structure (Li Hongxing, HX-group, BUSEFAL, 33(1987), 31-37). In a similar way, a ring structure has been upgraded to a ring structure defined on the power set, obtaining the notion of HX-ring (Li Hongxing, HX-ring, BUSEFAL, 34(1988), 3-8).

Consider $(R, +, \cdot)$ to be a ring and let $\mathcal{P}^*(R) = \mathcal{P}(R) \setminus \{\emptyset\}$, where $\mathcal{P}(R)$ denotes the power set of R. Define then on $\mathcal{P}^*(R)$ the operations:

$$\begin{array}{rcl} A + B & = & \{a + b \mid a \in A, b \in B\} \\ A \cdot B & = & \{a \cdot b \mid a \in A, b \in B\}. \end{array}$$

It is clear that $(\mathcal{P}^*(R), +)$ and $(\mathcal{P}^*(R), \cdot)$ are semigroups, but the structure $((\mathcal{P}^*(R), +, \cdot)$ is not a ring since only the inclusive distributive low

$$A \cdot (B + C) \subset A \cdot B + A \cdot C, \;\; (B + C) \cdot A \subset B \cdot A + C \cdot A$$

holds. A nonempty subset $\mathcal{R}$ of $\mathcal{P}^*(R)$ is called an *HX-ring* on R if $(\mathcal{R}, +, \cdot)$ forms a ring.

This notion can be more extended to the one of *power ring* in the following way. Let $\mathcal{R}$ be a nonempty subset of $(\mathcal{P}^*(R)$ endowed with two operations defined by

$$\begin{array}{rcl} A + B & = & \{a + b \mid a \in A, b \in B\} \\ A \circ B & \supseteq & A \cdot B = \{a \cdot b \mid a \in A, b \in B\}. \end{array}$$

If $(\mathcal{R}, +, \circ)$ forms a ring, then $\mathcal{R}$ is called a *power ring* on R. Clearly, all HX-rings are power rings on R. We provide a systematic comparison between the two significant subset -based algebraic structures: power rings vs. HX-rings. We first review HX-rings, focusing on their classification based on the properties of its zero element. Regular, normal and uniform HX-rings will be recalled and characterized as quotient sets under specific conditions. Similar properties will be then trasfered to power rings.

From Regular Relation to Quotient Hypergroups

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Abstract

A fundamental aspect of algebraic structures is that congruence relations determine quotient constructions. For a group G, congruences are in one-to-one correspondence with normal subgroups, given by the equivalence class of the identity element, and yield quotient groups of the form G/N. For a semigroup S, congruences are in one-to-one correspondence with quotient semigroups S/σ, where the equivalence classes define the induced operation. In the framework of hypercompositional structures, the natural extension of the notion of congruence is that of a regular relation. It is well known that a quotient (semi)hypergroup can be constructed as the quotient set modulo a regular relation. In this work, we present concrete classes of hypergroups in which regular relations are obtained by exploiting suitable subhypergroups. The corresponding quotient sets give rise to quotient hypergroups.

AI Tools and the Exploration of Hypercompositional Structures in Algebra

Charalampos Tsitouras

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Abstract

The proliferation of artificial intelligence tools provides novel methods for analyzing algebraic systems defined by hypercompositions. This paper investigates algorithmic approaches for classifying hypercompositional structures, utilizing symbolic computation environments. Specific attention is directed to the automated generation of Mathematica modules through natural language instructions provided to large language models. The derived computational packages facilitate the exact verification of H_v-groups, the evaluation of distributive properties within finite hyperfields, and the deterministic extraction of closed subhypergroups. The translation of mathematical axioms into precise prompts yields the exact algorithms required for structural evaluation.